During the present month her high altitude has rendered her surprisingly bright as a daylight object, and in strong contrast with her feebleness when I saw her at the end of February.

Bristol, May 30, 1876.

On the Atmosphere of Venus.
By E. Neison, Esq.

In the *Astronomische Nachrichten* for 1849 (No. 679, vol. xxix. p. 107) are published some observations, by Clausen and Mädler, of the prolongation of the cusps of *Venus* when the planet was near conjunction, and which showed that the Sun's rays underwent a considerable refraction from the atmosphere of the planet. The extent of the prolongation of the cusps of *Venus* was measured with the great Dorpat telescope, and from these measures the amount of the horizontal refraction exerted by the atmosphere of *Venus* was ascertained by means of formula which was given.

This formula was as follows:—

Let V = the angular distance between the centres of the Sun and *Venus*;

x = the prolongation of the cusps of the planet;

S = the semi-diameter of the Sun;

ϕ_1 = the radius-vector of *Venus*.

Then the horizontal refraction ζ_o of the atmosphere of *Venus* was found from the equation

$$\zeta_o = \frac{1}{2}\left(\psi - \frac{S}{\phi_1}\right),$$

where

$$\sin\psi = \sin V \sin x.$$

By this method Mädler found the horizontal refraction of the atmosphere of *Venus* to be 43′·7, or about $\frac{1}{6}$th greater than the Earth's. This result Mädler subsequently sent to the Astronomer Royal, who communicated an account of the same to the Royal Astronomical Society, and it appeared, in due course, in the *Monthly Notices* (vol. xviii. p. 320).

Lately this formula has been employed by Professor C. S. Lyman, of Sheffield, United States, to reduce similar measures of the prolongation of the cusps of *Venus*, made with a fine 9-in. refractor during 1866 and 1874. By its means Professor Lyman found from his observations 45′·3 and 44′·5 for the horizontal refraction of the atmosphere of *Venus*.

A note by Mr. R. A. Proctor, in the *Astronomical Register* (October 1875), induced me, as soon as I had the leisure, to

examine this formula of Mädler's, when it was at once apparent that as it had been employed by Mädler and Lyman it was defective. Instead of the angle V or the angular distance between the centres of the Sun and *Venus* as seen from the Earth which they had used, they should have employed the supplement of the angle between the Sun and Earth as seen from *Venus*. In consequence of this error the value which they have deduced from their observations for the horizontal refraction of the atmosphere of *Venus* is incorrect.

Retaining the notation above, and for additional accuracy putting

S' = the semi-diameter of *Venus*,

ϕ = the radius-vector of the Earth,

then the proper equation for finding the horizontal refraction is

$$\zeta_0 = \frac{\phi}{2\phi_1}\{(V + S' \sin x) \sin x - S\}.$$

The above expression is easily found, for if in the triangle formed by the Sun, Earth, and extremity of the prolongation of the cusp of *Venus*, we put θ for the angle at the Earth, and if ψ represents the apparent deflection of the solar rays, or the supplement of the angle at *Venus*, then

$$\frac{\sin \psi}{\phi} = \frac{\sin \theta}{\phi_1}.$$

It is easy to see that from this definition we have

$$\theta = V + S' \sin x,$$

and

$$\psi \sin x = 2\zeta_1 + \frac{\phi}{\phi_1} S.$$

It only remains to replace the ratio of the sines by the ratio of the angles, which is permissible, the angles being very small, and the above equation results at once.

Applying this formula to the observations given by Mädler in the *Astronomische Nachrichten* and the values for the horizontal refraction, which results are

$60'{\cdot}12$, $54'{\cdot}33$, $47'{\cdot}70$, $49'{\cdot}72$, $49'{\cdot}97$, $58'{\cdot}73$, and $60'{\cdot}44$;

the mean value is $54'{\cdot}43$, or $54'{\cdot}26''$, and a probable error of $1'{\cdot}38$.

The four observations by Lyman, made during 1874, are probably far superior in accuracy to Mädler's, and were made under better conditions. They are each the result of a number of repeated measures, and give for the horizontal refraction the values

$56'{\cdot}34$, $51'{\cdot}54$, $54'{\cdot}63$, and $51'{\cdot}49$;

the mean value is 53′·50 or 53′·30″, agreeing closely with the result obtained by Mädler.

The mean between these two results is 54′·65 or 54′·39″, and this value is probably within one minute of arc of the true value of the horizontal refraction of the atmosphere of *Venus*. The details of the measures made by Lyman in 1866 not having been published, it has not been possible to reduce these; considering, however, the nature of the error in the formula by which they were reduced by Lyman, the mean of these observations cannot differ much from the value 54′·7, a result well in accord with the other two series.

It appears, therefore, that the value found from their observations by Mädler and Lyman, for the horizontal refraction of the atmosphere of *Venus*, must be increased from 44′·5 to 54′·65.

From the value of the horizontal refraction of the atmosphere it is easy to deduce the surface density of the same. The method of effecting this has been already given (*Monthly Notices*, vol. xxxv., p. 331). Supposing the surface density equal to that of the Earth's, the horizontal refraction which it would exert could not differ much from 28′·53″. Consequently, the value found above for the amount of the horizontal refraction shows that the density at the surface of *Venus* of its atmosphere cannot differ much from being 1·892 times that of the Earth's, or nearly twice as dense. This is entirely in accord with the phenomenon presented by the planet, which indicates the presence of an atmosphere materially denser than that of the Earth's.

Repeated observations have shown that when *Venus* is near the Sun the entire planet can be seen as a dark circular disk. This appearance may arise either from the planet being seen projected against a bright background—such as, for example, the solar corona—or else from being surrounded by a faint ring of light due to the refraction of the solar rays by the atmosphere. Reversing the formulæ already given, it is easily found that the maximum distance from the Sun at which the planet's atmosphere has power to refract the solar rays so as to form a complete ring of light around the planet is

$$V = 2\frac{\phi^1}{\phi}\zeta_0 + (S - S').$$

The greatest value this can have is 1° 37′. But the ring of light would be sufficiently near completeness at a distance of even 2° to present the same appearance. From the natural tendency of the eye to supply the missing portion of a faint ring of light, it is very probable that, except in very careful observations, even at a distance of 5° from the Sun, the prolongation of the cusps of the planet (about 20°) would be sufficient to make it present the appearance of a circular dark disk on a brighter background.